COBB-DOUGLAS PRODUCTION FUNCTION

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- Many economists have studied the actual production functions and have used statistical methods to find out the relation between changes in physical inputs and physical outputs.
- A most famous empirical production function found out by statistical methods was Cobb-Douglas production function.

  (CW.COBB and PAUL H. DOUGLAS)
- It is a linear homogeneous production function takes into account two inputs labour and capital.
- The function take the following mathematical form,

$$Q = AL^{\propto} K^{\beta}$$

Where,

Q is the manufacturing output.

L is the quantity of labour employed.

K is the quantity of capital employed.

A is a positive constant.

α and β are positive fractions, and $\alpha + \beta = 1$

Here the efficiency in the organization of factors of production is measured by constant A.

- Economists have recently become greatly interested in Cobb Douglas production function because, it is consistent with constant return to scale.
- Thus, when the input of labour and capital are increased by a constant (say ‘g’),output ‘Q’ also increase by ‘g’ amount.

PROPERTIES OF COBB-DOUGLAS PRODUCTION FUNCTION

The C-D production function have several features and properties, that makes it so popular and useful…

The important properties are…

PROPERTIES

- Being log linear form the function is easy to handle.

$$Q = AL^{\propto} K^{\beta}$$

In logarithmic form the function can be written as

Log Q = log ($AL^{\propto} K^{\beta}$)

$$= LogA + \text{Log } L^{\propto} + \text{Log } K^{\beta}$$

$$= LogA + \alpha \text{ Log } L + \beta \text{ Log K}$$

- Since α+β= 1, In some special cases the function can be written as,

$$Q = AL^{\propto} K^{1-\alpha}$$

In this special case the function shows constant return to scale.

That is, when the inputs of labour and capital are increased by a constant “g” the output Q also increase by “g” amount.

Thus, α+β measure the return to scale.

If α+β= 1 Constant return to scale

α+β> 1 increasing return to scale

α+β< 1 Decreasing return to scale

- The function yield diminishing return to scale to each input (labour and capital).

When, Q = $AL^{\alpha} K^{\beta}$, the marginal product of labour is

$$MP_L = \frac{\partial Q}{\partial L} = \alpha AL^{\alpha-1} K^{\beta}$$

The second derivative of output with respect to labour is,

$$\frac{\partial Q^2}{\partial L^2} = (\alpha\text{-}1)\ AL^{\alpha-2}\ \alpha K^{\beta}$$

Since α is a positive fraction with a value less than one, α-1 is negative. Thus, the rate of change in MP_L is also negative and MP_L declines. This means, the function shows diminishing return to scale to labour.

Similarly, the marginal product of capital

$$\text{MP}_K = \frac{\partial Q}{\partial K} = \beta\, AK^{\beta-1}L^{\propto}$$

Second derivative output with respect to capital is,

$$\frac{\partial Q^2}{\partial K^2} = (\beta\text{-}1)\, AK^{\beta-2}\beta L^{\propto}$$

Since β is also a positive fraction with a value less than one, β-1 is negative. Thus, the rate of change in MP_K is also negative. This means, the function shows diminishing return to scale to factor capital.

- The exponents of labour and capital in Cobb-Douglas production function (α and β) measure the output elasticity of labour and capital. ie; α shows the output elasticity of labour and β shows the output elasticity of capital.

The output elasticity is defined as, the ratio of relative change in output over a relative change in input.

The output elasticity of labour is,

$$\frac{\partial Q}{Q} \div \frac{\partial L}{L} = \quad \frac{\partial Q}{Q} \times \frac{L}{\partial L} = \quad \frac{\partial Q}{\partial L} \times \frac{L}{Q} = \quad \frac{\alpha\, AL^{\alpha -1}\; K^{\beta} \times L}{AL^{\alpha}\; K^{\beta}}$$

$$= \frac{\alpha\, AL^{\alpha}\; K^{\beta}}{AL^{\alpha}\; K^{\beta}} = \alpha$$

- Thus, the exponent α in Cobb-Douglas production measures the output elasticity of labour.

Similierly, The output elasticity capital is

$$\frac{\partial Q}{Q} \div \frac{\partial K}{K} = \frac{\partial Q}{Q} \times \frac{K}{\partial K} = \frac{\partial Q}{\partial K} \times \frac{K}{Q}$$

$$= \frac{\beta\ AK^{\beta-1} L^{\propto} \times K}{AL^{\propto}\ K^{\beta}}$$

$$= \frac{\beta\ AL^{\propto}\ K^{\beta}}{AL^{\propto}\ K^{\beta}} = \beta$$

Thus, the exponent β in Cobb-Douglas production measures the output elasticity of capital.

- In Cobb-Douglas production function the exponents α and β shows the relative distributive share of labour and capital. ie; α shows the relative distributive share of labour and β shows the relative distributive share of capital.

Relative distributive share of labour is,

$$\frac{\partial Q}{\partial L} \times \frac{L}{Q} = \frac{\alpha\, AL^{\alpha-1}\; K^{\beta} \times L}{AL^{\alpha}\; K^{\beta}}$$

$$= \frac{\alpha\, AL^{\alpha}\; K^{\beta}}{AL^{\alpha}\; K^{\beta}} = \alpha$$

Thus, α shows relative distributive share of labour.

Similarly,

Relative distributive share of capital is

$$\frac{\partial Q}{\partial K} \times \frac{K}{Q} = \frac{\beta\, AK^{\beta-1}L^{\propto} \times K}{AL^{\propto}\; K^{\beta}}$$

$$= \frac{\beta\, AL^{\propto}\, K^{\beta}}{AL^{\propto}\; K^{\beta}} = \beta$$

Thus, β shows relative distributive share of capital.

- For Cobb-Douglas production function elasticity of substitution between two factors (L and K) equals to unity.

Ie; Elasticity of substitution $\sigma = 1$

$$Q = AL^{\propto} K^{\beta}$$

$$\text{MPL} = \alpha \; AL^{\propto -1} \; K^{\beta}$$

$$= \alpha \; AL^{\propto} \; K^{\beta} \times L^{-1} = \alpha Q L^{-1}$$

$$= \frac{\alpha \, Q}{L}$$

$$\text{MP}_K = \beta \, AK^{\beta-1} L^{\propto}$$

$$= \beta \, AK^{\beta} L^{\propto} \times K^{-1} = \beta Q K^{-1} = \frac{\beta Q}{K}$$

We have $\text{MRTS}_{LK} = \frac{MPL}{MPK}$

$$= \frac{\alpha Q/L}{\beta Q/K} = \frac{\alpha}{\beta} \times \frac{K}{L}$$

Elasticity of substitution,

$$\sigma = \frac{d(K/L)}{K/L} / d(\text{MRTS}_{LK}) / \text{MRTS}_{LK}$$

Substitute equations,

$$\sigma = \frac{d\left(\frac{K}{L}\right)}{\frac{k}{L}} / d\left(\frac{\alpha}{\beta} \times \frac{K}{L}\right) / \left(\frac{\alpha}{\beta} \times \frac{K}{L}\right)$$

$$\sigma = \frac{d\left(\frac{K}{L}\right)}{\frac{k}{L}} \times \left(\frac{\alpha}{\beta} \times \frac{K}{L}\right) / d\left(\frac{\alpha}{\beta} \times \frac{K}{L}\right) = \frac{d\left(\frac{K}{L}\right) \times \frac{\alpha}{\beta}}{d\left(\frac{\alpha}{\beta} \times \frac{k}{L}\right)}$$

$$\sigma = 1$$

- Cobb-Douglas production function can be extended to include more than two variable.

For example, agricultural production depends not only on labour and capital used, but also the use of other inputs such as, land, irrigation, fertilizers etc.

Then the function will be,

$$Q = AL^{a}\ K^{b1}\,D^{b2}\ G^{b3}\,F^{b4}$$

Where, D stands for land,

- G - irrigation,
- F – fertilizers
- a,b1,b2,b3,b4 are the exponents of these factors.

- The marginal product of labour and capital depends only on the quantities of labour and capital used in the production

Marginal product of labour,

$$MP_L = \frac{\partial Q}{\partial L} = \alpha\, AL^{\alpha-1}\, K^{\beta}$$

$$= \alpha\,(AL^{\alpha} K^{\beta})\, L^{-1}$$

$$= \alpha\, \frac{Q}{L}$$

$$= \alpha(AP_L)$$

Thus, the MP_L depend on the quantity of labour used in production.

Similarly,

The marginal product of capital

$$MP_K = \frac{\partial Q}{\partial K} = \beta\, AK^{\beta-1}L^{\propto}$$

$$= \beta\,(AK^{\beta}L^{\propto})\,K^{-1}$$

$$= \beta\,\frac{Q}{K}$$

$$= \beta\,(AP_K)$$

This shows that marginal product of capital is a function of capital used.

THANK YOU